Belridge Secondary College

Mathematics Specialist Unit 4

Test 4: Calculus **solutions**

**Name:** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Section One – calculator-free**

**1. [2, 2, 2, 4 = 10 marks]**

Determine the following definite integrals; using the substitution, when given.

1. ✓

✓

1. using the substitution,

✓

✓

1. d using the substitution,

d ✓

✓

1. d d

✓

✓

✓

✓

**2. [3, 3 = 6 marks]**

1. Express in the form

✓

Let

Let ✓

✓

1. Hence determine expressing your answer as a single logarithm.

)

✓

✓

✓

**3. [4, 4 = 8 marks]**

1. Using the substitution show that and state the values of

LHS ✓

✓

✓

RHS with and ✓

1. Hence, or otherwise, evaluate exactly.

✓

✓

✓

✓

**End of Section One**

**Section Two – calculator-assumed**

**4. [3, 5 = 8 marks]**

The graph of the curve of equation is shown below. The line of equation is tangent to the curve at point P. The shaded region is enclosed by the curve, the tangent line and the axis.

****

P

1. Find the co-ordinates of point P and the intercepts of the curve and the tangent line.

and intercepts at ✓✓✓

1. Find an integral expressionfor the area of this region in terms of
2. horizontal elementary slices

✓

✓

Area ✓

1. vertical elementary slices

✓

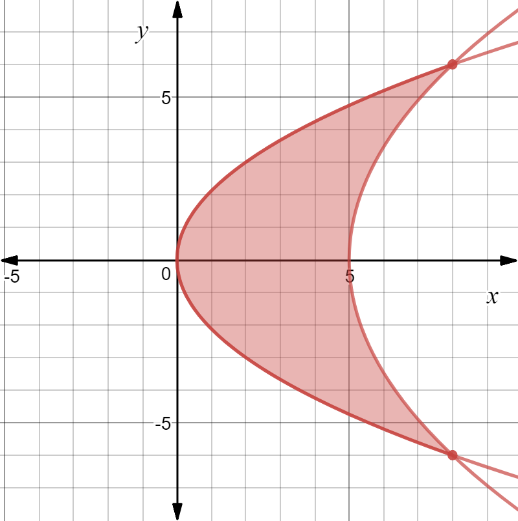
Area

**5. [4, 4 = 8 marks]**

For the curves with equations and determine the volume of the solid formed when the region enclosed by the two curves is

1. rotated about the

and ✓



✓

✓

1. rotated about the axis

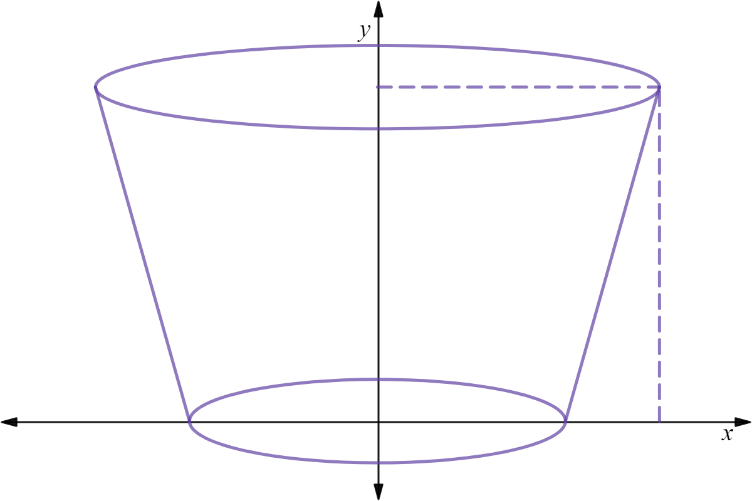
and ✓

✓✓

✓

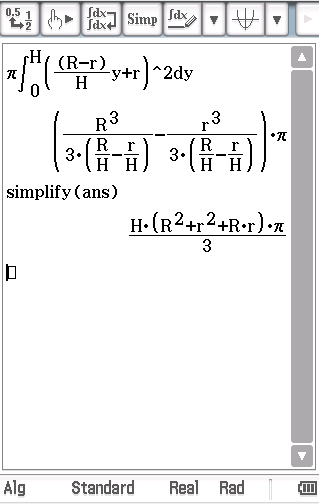
**6. [5 marks]**

A bucket has a circular base of radius cm and a circular rim of radius cm. The perpendicular height of the bucket is cm. Use calculus techniques to determine, in simplest form, the volume of water, in terms of this bucket will hold when full.

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gradient of edge of bucket ✓

equation of line ✓✓



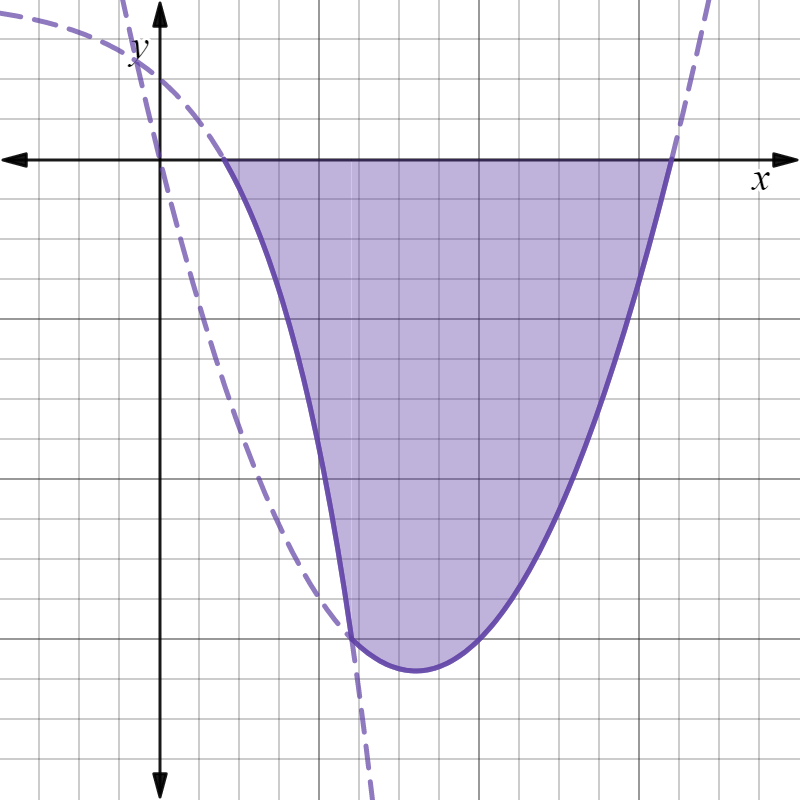
✓

✓

**7. [5 marks]**

Below is shown the uniform cross-section of a drainage canal (shaded), the bed of which is described by curves; and , where and are measured in metres and the axis represents the water level. If the water is flowing at the rate of then how many kilolitres (to the nearest kL) of water will pass by in two minutes.

Note: 1 m3 = 1kL

****

and

Area ✓✓

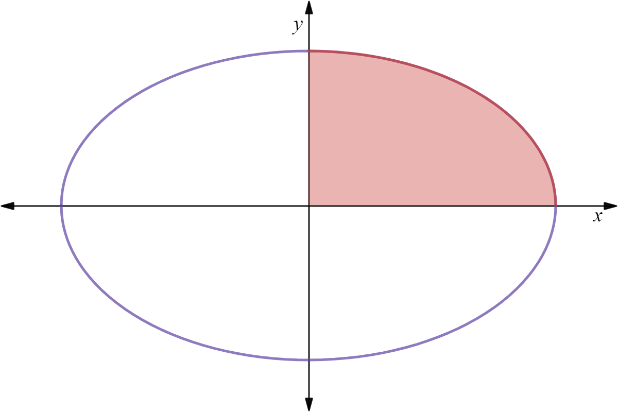
✓

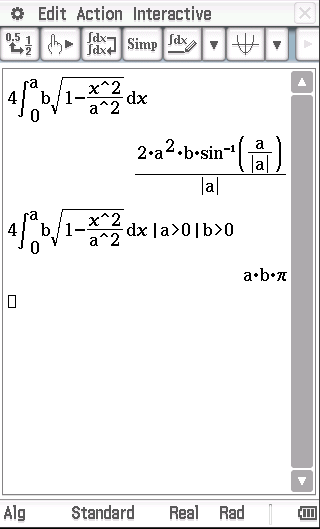
Volume of water ✓

to the nearest ✓

**8. [3, 2 = 5 marks]**

The ellipse shown has equation with .

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Use a suitable integral to determine, in terms of ,

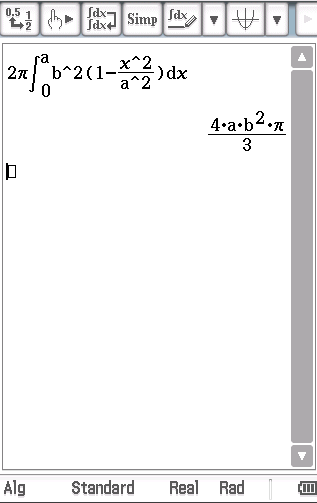
1. the area of the ellipse.

✓

Area ✓

✓

1. the volume of the ellipsoid formed by rotation of the ellipse about the axis.



✓

✓

**End of Section Two**